Small Business Environment and Investment Climate*

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Abstract

One of the main problems of the Russian economy in transition is the segmentation of its capital and labor markets. While there are many barriers to labor mobility, it is less clear why the capital does not come to regions with low wages and high unemployment. A possible explanation of this fact could be that the state of the regional labor market may itself be endogenous to poor small business development and strategies of regional authorities.

This paper considers a game between a regional governor and an outside investor taking into account the interactions in the regional labor market. The investor wants to invest in a large company in the region. The investment raises labor productivity, so the investor wants to fire excess workers. Redundant workers can try to open small firms or become unemployed. If the level of small business environment is high then almost all fired workers can open small firms and the governor welcomes investment. But if the level of small business environment is low, the investor has to pay the region for raising unemployment or to maintain the excess employment; in both cases investment is discouraged. The effects of small business environment on the governor's attitude to investors may overweigh the upward pressure on wages due to booming small business that reduces investors' profits.

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1 Introduction

One of the main problems of the Russian economy in transition is the regional segmentation of its product, labor, and capital markets. The barriers to interregional trade in goods and services can be explained by provincial protectionism

(Berkovitz (2000), Sonin (2003)). There are also multiple barriers to geographical labor mobility: financial constraints, underdevelopment of housing market and sheer distance (Friebel and Guriev (2000), Andrienko and Guriev (2003)). It is less clear, though, why the capital mobility does not make up for the lack of mobility of other factors. If the labor is locked in a region with low wages and high unemployment, such a region should attract capital investment. However, the data on Russian regional show that investment per capita (a) differs substantially across regions (see Appendix 1) and (b) is not related to the regional labor market conditions.

In this paper we argue that the analysis of interregional capital mobility is incomplete without taking into account incentives of regional authorities and the small business environment. Economic theory suggests that better opportunities for self-employment and small business development increase wages exert an upward pressure on wages in large firms in the regions hence increasing production costs and decreasing incentives for outside investors. However, one should also consider the response of regional authorities. In modern Russia, a regional governor has a lot of discretion over regulatory burden on the investors. The governor cares about elections and therefore wants to keep unemployment low. Therefore governor can use his discretion to make sure that a profit-maximizing investor does not fire excess workers.

This paper analyzes the influence of small business environment on the investment in large enterprises. We study a general equilibrium model of a regional economy and focus on a game between an investor and the regional governor. The setup is as follows. The investor wants to acquire a large enterprise and restructure it. Investment increases labor productivity. We assume that external demand for the enterprise's output is not elastic, so profit maximization requires firing excess workers. The redundant workers can try to open small firms. The expected return of small business depends on the aggregate regional consumer demand and the level of small business environment. In this paper, the level of small business environment is considered exogenous. If the level of small business environment is high then almost all fired workers will open small firms (increased income of individuals raises demand for production of small firms) and the governor will welcome the investor with lower taxes. But if the level of business environment is low, the governor and the investor will bargain about the level of excess employment, and if the outcome of their negotiations may increase investor's costs substantially. On the other hand, the equilibrium in the local labor market produces an opposite effect of small business on investment in large firms: better self-employment opportunities raise the reservation wages of industrial workers and therefore reduce investor's profits. In this paper, we analyze both effects, and study the impact of policies on investment, regional fiscal revenues, and unemployment in regions with different small business environment.

In this paper, the small business environment is exogenous. It is done for simplicity. Apparently, regional governments can affect small business environment in many ways. Slinko, Yakovlev and Zhuravskaya (2003) give an example of such kind of influence: regional authorities can directly affect small business environment, by changing costs of registration, certification, inspections,licensing, and leasing premises. Zhuravskaya (2000) describes the model where a mayor can choose the level of public goods provision and the level of regulation of private business. The high level of public goods provision and the absence of regulation lead to the high level of social welfare, but the mayor has incentives to impose regulation on private business: the more regulation means the higher opportunity to receive more bribes because bribes are offered in exchange for relief from regulations. Gehlbach (2003a) states that the government is interested in developing of infrastructure only for large enterprises, because large enterprises are typically more taxable than small firms — small firms are more likely to deal in cash (and thus find it easier to hide revenues), also, large firms may be favored because they provide politically important employment.

There are several studies in the related areas. Sonin (2003) analyzes the model where the governor can affect the level of market competition in the region, and give exemptions from federal taxes. The low level of market competition and tax exemptions can increase the rent of particular firms, so the governor can collect more bribes, sharing the rent with these firms. The more concentrated is the industrial structure of the region, the higher is the tax protection of regional authorities. Schleifer and Vishny (1994) analyze the negotiations between the governor and the manager of the firm concerning bargaining process on excess employment. Though the excess employment reduces the firm's profit, it decreases unemployment in the region, so it helps to increase the chances of the governor to win in future elections. Berkowitz and Li (2000) describes the model where the authorities can choose the size of investment in public infrastructure and the tax rate. Because fiscal institutions in transition economies are often underdeveloped, each level of the government is unable to commit to a tax policy, and therefore exercises its discretion by setting its tax rate after investors have sunk their investments. Since each agency sets up tax rates ex-post and ignores the negative impact of an increase in its own tax rate on the other agencies and therefore tends to set an excessively high tax rate, investors ex-ante abstain from making investment, so authorities do not have incentives to invest money in public infrastructure. McMillan and Woodruff (2002) analyze the importance of entrepreneurship development in the process of formation of market institutions. Basareva (2002) analyzes convergence and divergence tendencies in small business development in Russian regions. The general conclusion from this paper is that small business sector is important part of the economy and it is an important for the development of the economy.

The rest of the paper is structured as follows. Section 2 develops the model of product market, small business sector, large enterprise, and labor market. Section 3 describes the equilibrium state of labor market. Section 4 analyzes comparative statics and implications of different policies of the governor, while Section 5 concludes.

2 The Model

There are two players: a governor and an investor. The utility function of the governor positively depends on the amount of regional budget revenues and negatively depends on the unemployment level:

$$U_{gov}(G, u) = G - \theta u \quad (1)$$

where G is the regional budget revenues , u is the level of unemployment.

The investor acquires a large company in the region and set new technology, which allows to increase productivity. The investor maximizes NPV.

2.1 Product Market

There are three kind of goods in economy: imported industrial goods, exported industrial goods (produces by the large industrial firm), and services supplied by small business. Households consume only imported industrial goods and services.

The demand for exported industrial goods is exogenous function $D(p)$. For simplicity, let the demand have constant elasticity:

$$D(p) = \frac{1}{p^{\varepsilon}}$$

The market for small business output is a monopolistic competition market. Households maximize Cobb-Douglas utility:

$$U_{cit}(Q_s, Q_r) = Q_s^{\mu} \cdot Q_r^{1-\mu}$$

where Q_s is consumption of the "small business" services, Q_r is consumption of the imported industrial goods.Q_s is the composite good and consists of the q_i(production of each small firms, Dixit-Stiglitz model):

$$Q_s = \left(\sum_{i=1}^{n} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where n is the number of small firms, σ is the elasticity coefficient. Let I be the income of consumer and consumer solves the following maximization problem:

$$U_{cit}(Q_s, Q_r) \longrightarrow \max \quad (2)$$

$$s.t. \quad \sum_{i=1}^{n} q_i p_i + P_r Q_r \leq I$$

The demand for services and imported industrial goods can be derived from (2).

2.2 Small Business

The marginal cost of production is constant and equal to c. The fixed cost of production is g, the random variable which is uniformly distributed on $[0..1]$. g reveals at the beginning of the firm's life and remains the same in persistency. Let λ be the "friendliness" coefficient. Since c, q_i, p_i are observable variables and g_i is an unobservable variable (it is private information of each entrepreneurs), bureaucrats (firemen, policemen, and etc.) can expropriate only the part of the observable income, and the expropriation is $(1-\lambda)(p_i-c)q_i$. We can also interpret λ as the level of the quality of small business environment (the development of courts, the quality of laws, the development of banking system, etc.). Thus, the profit of each small firm is equal to:

$$\pi_i = \lambda (p_i - c) q_i - g_i \rightarrow \max_{p_i, q_i} \tag{3}$$

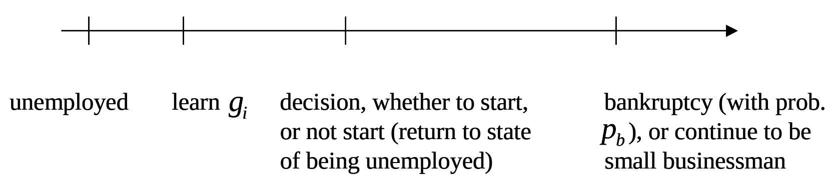
By solving the problems (2) - (3) we can find the equilibrium:

$$p_i = \frac{\sigma}{\sigma - 1} \cdot c, \quad q_i = \frac{\mu \cdot I \cdot (\sigma - 1)}{c \cdot \sigma \cdot n}$$

So the profit of each firm in equilibrium is equal to:

$$\pi_i = \lambda \frac{\mu \cdot I}{\sigma \cdot n} - g_i \tag{4}$$

The timing for a small businessman is the following:



2.3 Large Enterprise

There is one large enterprise in the region. All goods, produced by this company, are exported outside the region. The investor acquires this enterprise, and set new technology. The timing is as follows. At the first stage the investors chooses the volume of investment in fixed capital K. At the second stage, the investment is sunk and the investor chooses the level of employment, L, maximizing the profit:

$$\Pi = f(K, L) \cdot p(f(K, L)) - wL \rightarrow \max_L \tag{5}$$

where $f(K, L)$ is the production function and $p(f(K, L))$ is the inverse demand function. Let the production function be the following:

$$f(K, L) = (K^a + L^a)^{\frac{1}{a}}$$

where a is some positive coefficient.

The investor chooses K, maximizing NPV of the entire project (for simplicity, let the wage remains the same in persistency):

$$\sum_{i=1}^{\infty} \delta^i \Pi(K, w) - K \to \max_K$$

where $\Pi(K, w)$ is the solution of maximization problem (5) and δ is the discount coefficient.

2.4 Labor Market

2.4.1 Labor Supply

All individuals in the city have three opportunities: to work at the large enterprise, to open a small business, or to be unemployed.

The number of people in the city is normalized to 1. The large enterprise employs α workers. People can either work at the large enterprise or be small businessmen. There are no other job opportunities. All individuals have equal opportunity to be hired or to open a small firm; there is no transaction cost (application cost or setup costs for small business). The unemployment benefit is equal to b. Therefore the total income of the individuals is equal to:

$$I = n \cdot \overline{\pi} + \alpha \cdot w + (1 - \alpha - n) \cdot b \tag{6}$$

where w is the wage of workers, n is the number of small firms, and $\overline{\pi}$ is the average profit of the small firms.

Let people find and lose jobs a la Poisson process. The small firms also become bankrupt according to Poisson process. Let V_i denote the "value" of being in state i, in other words, it is the expected discounted value of lifetime utility. Since we are focusing on steady states, V_i's are constant over time.

Firstly, let's look at the decision process whether to open firm or continue to be unemployed. If somebody decides to open a small firm he looks at his level of g_i. If $g_i \leq \overline{g}$ he opens the firm, otherwise he continues to be unemployed. Obviously the level of $\overline{g}$ can be determined from equality of the firm's profit with fixed cost and expected return of the state "unemployed":

$$\lambda \frac{\mu \cdot I}{\sigma \cdot n} - \overline{g} = rV_u \tag{7}$$

where r is the discount rate, V_u is the value of being unemployed. Since g is uniformly distributed on $[0..1]$ we can find the average profit $\overline{\pi}$. Since $E(g|g < \overline{g}) = \overline{g}/2$, we can derive:

$$\overline{\pi} = \lambda \frac{\mu \cdot I}{\sigma \cdot n} - \frac{\overline{g}}{2} \tag{8}$$

Let p_b be the exogenous "probability" to become bankrupt, then the expected return from the state "small businessman" must be equal to "dividend" $\overline{\pi}$, plus expected capital losses:

$$rV_s = \overline{\pi} + p_b \cdot (V_u - V_s) \tag{9}$$

where V_s is the "value" of being small businessman. If an unemployed decides to open a small firm, then the probability of success is equal to $F(\overline{g}) = \overline{g}$, and we can write the following equation:

$$rV_u = b + \overline{g} \cdot (V_s - V_u) \tag{10}$$

We can derive the same for those who decide to become workers:

$$rV_u = b + p_e \cdot (V_e - V_u) \tag{11}$$

where p_e is the "probability" to be hired and V_e is the value of being a worker.

Finally, the equation of being in state "employed" is the following:

$$rV_e = w + p_f \cdot (V_u - V_e) \tag{12}$$

where p_f is the exogenous "probability" of being fired.

The difference between V_e and V_u is equal to d, it can be interpreted as a rent which can be collected by workers (e.g. we can derive this rent from no shirking condition):

$$V_e = V_u + d \tag{13}$$

Let N_1 be the number of applicants to the factory and N_2 be the number of people who decide to try to open a firm. Since there are no application costs or setup costs for business, all unemployed either apply to the factory or try to open a firm. Therefore $N_1 + N_2$ is equal to the total number of unemployed:

$$N_1 + N_2 = 1 - n - \alpha \tag{14}$$

In the steady state, obviously the number of bankrupt firms must be equal to the number of successful "applicants" to small business. Since $F(\overline{g}) \cdot N_2$ is the number of successful "applicants" to small business, $p_b \cdot n$ is the number of bankrupts each period, and $F(\overline{g}) = \overline{g}$, we can derive:

$$N_2 = \frac{p_b \cdot n}{\overline{g}} \tag{15}$$

The same is true for workers who apply to the factory; the number of new entrants must be equal to the number of fired workers. Therefore we can write the following equation:

$$p_e = \frac{\alpha \cdot p_f}{N_1} = \frac{\alpha \cdot p_f}{1 - \alpha - n - \frac{p_b \cdot n}{\overline{g}}} \tag{16}$$

2.4.2 Labor Demand

The labor demand function can be derived from the F.O.C. of profit-maximization problem:

$$\Pi = f(K,L) \cdot p(f(K,L)) - wL \to \max_L$$

where $f(K,L) = (K^a + L^a)^{\frac{1}{a}}$ and $p(f(K,L)) = \frac{1}{f(K,L)^{\frac{1}{\varepsilon}}}$.

3 Equilibrium

From the equations (6) - (16) and labor demand function we can derive the equilibrium level of wages, employment, the number of small firms, etc. Also we can analyze the impact of investment (investor increases K) and the different policies of the governor for stimulating employment situation. Since the investor pays more taxes to the budget, the governors can spend this money in different ways (he can give transfers to households, give subsidies to small firms or require hoarding labor).

3.1 Labor Supply

For simplicity, let's assume that $b = 0$. The industrial labor supply curve can be derived from the equations (6) - (16) (see Appendix 2 for details):

$$L_{is}(w) = \frac{\overline{g}^2 \left[\overline{g}(\sigma - \lambda\mu) + p_b(2\sigma - \lambda\mu)\right]}{\left[\overline{g}(\sigma - \lambda\mu) + p_b(2\sigma - \lambda\mu)\right]\left(\overline{g}^2 + 2p_b p_f \cdot d\right) + 2p_b(\overline{g} + p_b)\lambda\mu \cdot w} \tag{17}$$

The level of $\overline{g}$ is equal to:

$$\overline{g}(w) = \sqrt{2(r + p_b)(w - (p_f + r)d)} \tag{18}$$

The number of small firms is equal to:

$$n_{sb}(w) = \frac{\overline{g}(w)}{\overline{g}(w) + p_b} - L_{is}(w)\left(\frac{\overline{g}(w)}{\overline{g}(w) + p_b}\right)\left(1 + \frac{p_f \cdot d}{w - (p_f + r)d}\right) \tag{19}$$

And if $r \to 0$ then (17) can be rewritten as:

$$L_{is}(w) = \left(1 - \frac{p_f d}{w}\right)\left(1 - \frac{\lambda\mu(\overline{g} + p_b)}{\sigma(\overline{g} + 2p_b)}\right) \tag{20}$$

Let's denote "total labor supply" function $L_s(w) = L_{is}(w) + n_{sb}(w)$:

$$L_s(w) = \frac{\overline{g}\,(w)}{\overline{g}\,(w) + p_b} + L_{is}\,(w)\left[1 - \left(\frac{\overline{g}\,(w)}{\overline{g}\,(w) + p_b}\right)\left(1 + \frac{p_f \cdot d}{w - (p_f + r)\,d}\right)\right] \quad (21)$$

It is also interesting to analyze how industrial labor supply and "total labor supply" (21) depends on λ.

It is easy to show that $L_{is}\,(w)$ is a decreasing function of λ (it is a quite obvious result and corresponds with economic intuition: the better small business environment is, the more people want to open firms, and the less people prefer to work in the industrial sector).

The situation with "total labor supply" is not so obvious.

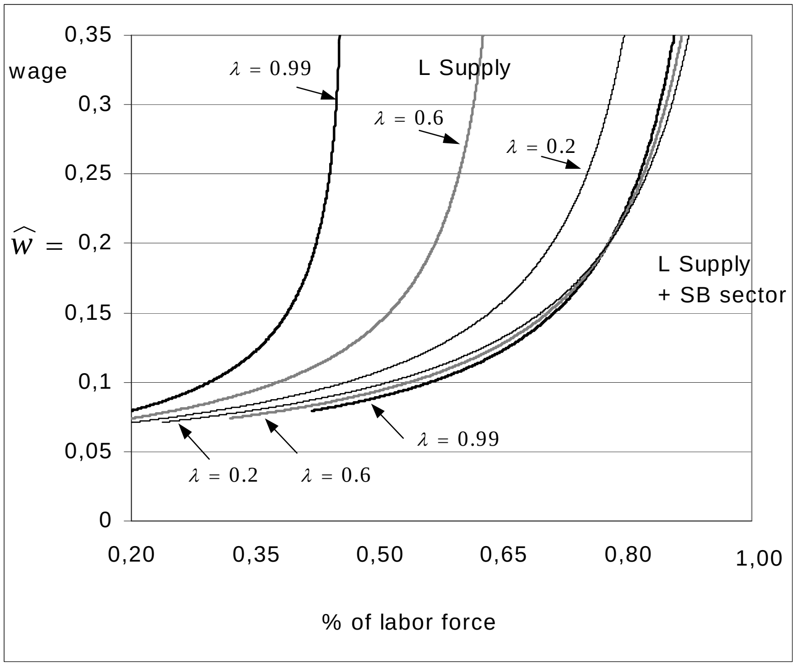
If $\left(\frac{\overline{g}(w)}{\overline{g}(w)+p_b}\right)\left(1 + \frac{p_f \cdot d}{w-(p_f+r)d}\right) > 1$ then a decrease in labor supply leads to an increase in "total labor supply" and vice versa. But if $\left(\frac{\overline{g}(w)}{\overline{g}(w)+p_b}\right)\left(1 + \frac{p_f \cdot d}{w-(p_f+r)d}\right) < 1$ then a decrease in labor supply leads to a decrease in "total labor supply".

Proposition 1.

There is $\widehat{w} = 2\,(r + p_b)\,\frac{(p_f d)^2}{p_b^2} + (p_f + r)\,d$ such as if $w < \widehat{w}$ then $L_s\,(w, \lambda)$ is an increasing function of λ, if $w > \widehat{w}$ then $L_s\,(w, \lambda)$ is a decreasing function of λ, and if $w = \widehat{w}$ then $L_s\,(w, \lambda)$ does not depend on λ. In other words if $w < \widehat{w}$ then for each $\lambda_1 < \lambda_2$, $L_s\,(w, \lambda_1) < L_s\,(w, \lambda_2)$, if $w > \widehat{w}$ then for each $\lambda_1 < \lambda_2$, $L_s\,(w, \lambda_1) > L_s\,(w, \lambda_2)$ and for each λ_1, λ_2 , $L_s\,(\widehat{w}, \lambda_1) \equiv L_s\,(\widehat{w}, \lambda_2)$.

Proof.

See Appendix 2.



The industrial labor supply and the total labor supply for different λs

Also we can note that the greater λ is, the steeper the "total labor supply" is.

If $r \to 0$ then $\frac{g(\widehat{w})}{2} = p_f \cdot d$, in other words, the "total labor supply" is constant in the point where the average fixed costs of small business sector are equal to expected "capital losses" of being in state employed.

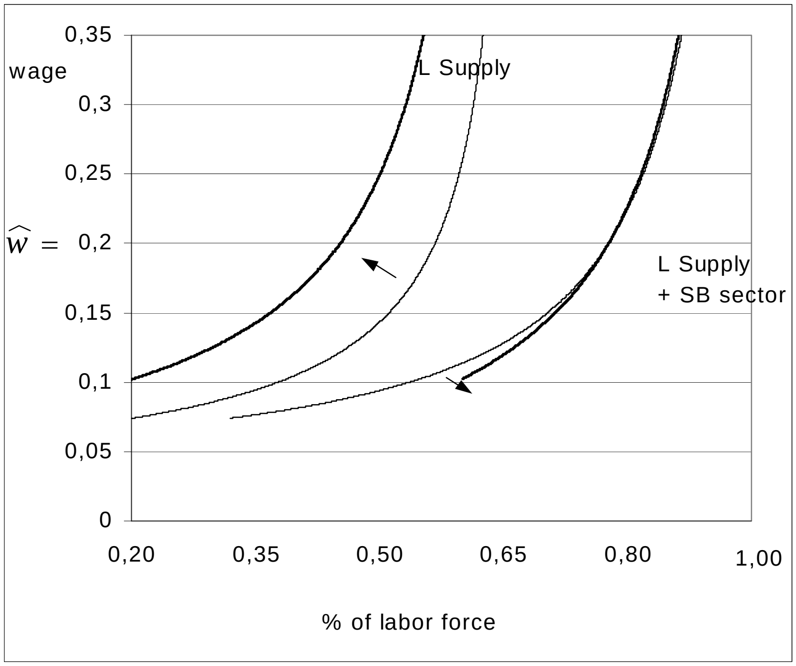
It is interesting to analyze how the situation changes if the total income increases by some ΔI (e.g. transfer from federal budget). The equation (17) becomes:

$$L_{is}(w) = \frac{\overline{g}^2 \left[\overline{g}(\sigma - \lambda\mu) + p_b(2\sigma - \lambda\mu)\right] - 2p_b(\overline{g} + p_b)\lambda\mu \cdot \Delta I}{\left[\overline{g}(\sigma - \lambda\mu) + p_b(2\sigma - \lambda\mu)\right]\left(\overline{g}^2 + 2p_b p_f \cdot d\right) + 2p_b(\overline{g} + p_b)\lambda\mu \cdot w} \tag{22}$$

And if $r \to 0$:

$$L_{is}(w) = \left(1 - \frac{p_f d}{w}\right)\left(1 - \frac{\lambda\mu(\overline{g} + p_b)}{\sigma(\overline{g} + 2p_b)}\right) - \frac{(\overline{g} + p_b)\lambda\mu \cdot \Delta I}{\sigma(\overline{g} + 2p_b)w} \tag{23}$$

This case is analogical the previous one: if $w = \widehat{w}$ the "total labor supply" is constant and does not depend on ΔI.

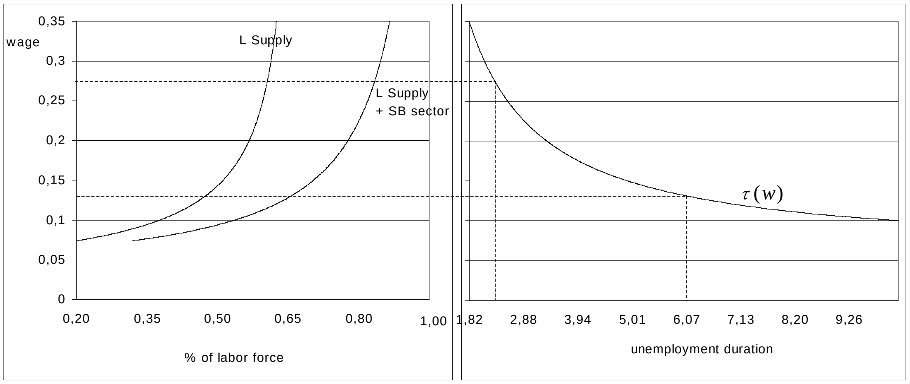


External transfer

It is interesting to note, that there is a multiplicative effect of ΔI. If the total income of individuals increases by one unit then the total profit of small business sector increase by $\frac{\frac{\lambda\mu}{\sigma}}{1-\frac{\lambda\mu}{\sigma}}$ or $\frac{\lambda\mu}{\sigma-\lambda\mu}$.

During our analysis we are going to concentrate mainly on the case $w < \widehat{w}$, because if $w > \widehat{w}$ then the improvement of small business environment or additional transfers to households lead to a decrease of total employment (a decrease in industrial employment is more than an increase in small business employment), so the policies for stimulating employment (additional demand for production of small business sector or improvement of small business environment) do not have positive effect on the employment situation.

We also can define the new function $\tau(w) = \frac{1-L_s(w)-n(w)}{p_f \cdot L_s(w)+p_b \cdot n(w)}$ - unemployment duration



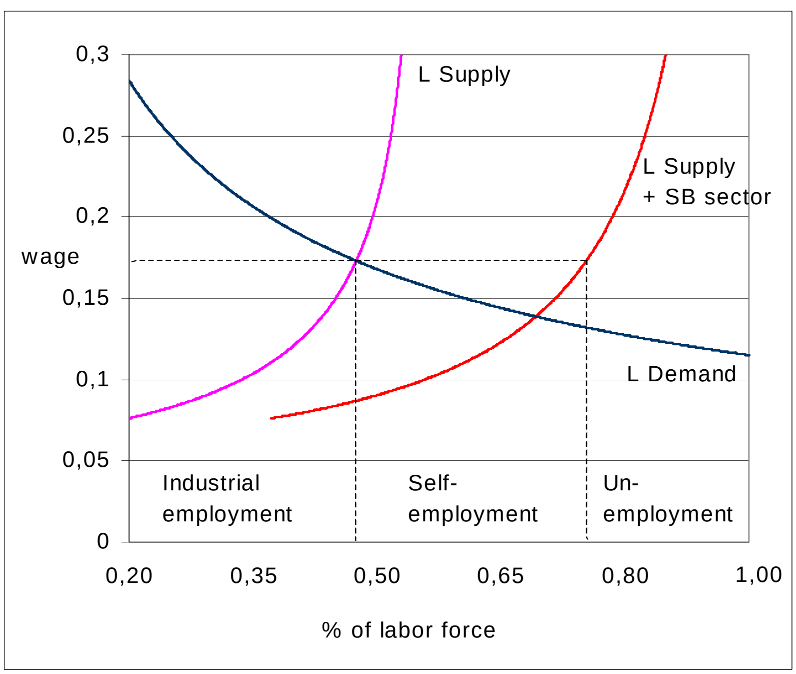
Labor supply and unemployment duration

3.2 Labor Demand and Equilibrium

From F.O.C. (5) we can derive the inverse labor demand function:

$$w(L_d) = \left(1 - \frac{1}{\varepsilon}\right)(K^a + L_d^a)^{\frac{1}{a}\left(1-\frac{1}{\varepsilon}\right)-1} L_d^{a-1} \qquad (24)$$

(24), (17) and (21) can be presented at L / w diagram:



Labor supply, demand and equilibrium

4 Comparative Statics and Policy Implications

After calculation of the equilibrium we can analyze the impact of the investment. The investor increases K from K_0 to K_1. It is interesting to analyze only the case of inelastic demand $\left(\varepsilon < \frac{1}{1-a}\right)$, because otherwise an increase of capital leads to an increase of labor demand and increase of employment, so in this case there is no conflict between the governor and the investor, and there is no space for analysis.

Let Π_0 be the profit in the original equilibrium (equilibrium with K_0):

$$\Pi_0 = \left(K_0^a + L_0^a\right)^{\frac{1}{a}\left(1-\frac{1}{\varepsilon}\right)} - w_0 L_0$$

Let's denote the new function $d\Pi\left(L\right)$ as the difference between Π_1 and Π_0:

$$d\Pi\left(L\right) = \left(\Pi_1\left(L\right) - \Pi_0\right)_+$$

where

$$\Pi_1\left(L\right) = \left(K_1^a + L^a\right)^{\frac{1}{a}\left(1-\frac{1}{\varepsilon}\right)} - w\left(L\right) L$$

and $w\left(L\right)$ is the wage which gives L as the optimal solution of (5):

$$w(L) = \left(1 - \frac{1}{\varepsilon}\right)\left(K_1^a + L^a\right)^{\frac{1}{a}\left(1-\frac{1}{\varepsilon}\right)-1} L^{a-1}$$

So, regional budget revenues increase by $t \cdot d\Pi\left(L\right)$, where t is the profit tax rate. The governor can spend additional revenues by different ways. This paper analyzes two ways: the governor gives transfers to households and gives subsidies to small business.

4.1 Transfers to Households

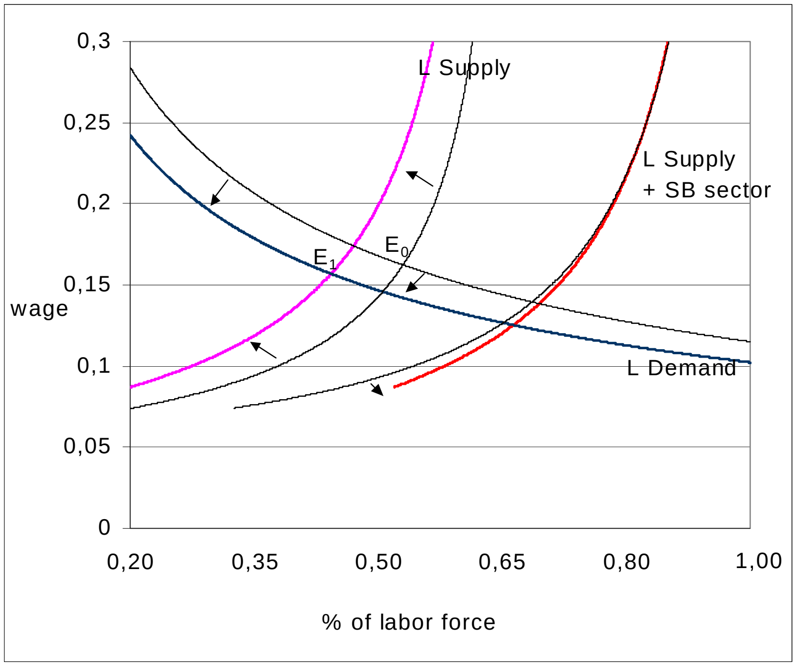
Transfers to households are equivalent to adding $t \cdot d\Pi\left(\alpha\right)$ to the total income:

$$I = n \cdot \overline{\pi} + \alpha \cdot w + t \cdot d\Pi\left(\alpha\right) \tag{25}$$

From (22) and (25) we can derive new industrial labor supply curve:

$$L_{is}\left(w\right) = \frac{\overline{g}^2\left[\overline{g}\left(\sigma - \lambda\mu\right) + p_b\left(2\sigma - \lambda\mu\right)\right] - 2p_b\left(\overline{g} + p_b\right)\lambda\mu \cdot t \cdot d\Pi\left(L_s\left(w\right)\right)}{\left[\overline{g}\left(\sigma - \lambda\mu\right) + p_b\left(2\sigma - \lambda\mu\right)\right]\left(\overline{g}^2 + 2p_b p_f \cdot d\right) + 2p_b\left(\overline{g} + p_b\right)\lambda\mu \cdot w}$$

Using the new equation for labor supply and labor demand, we can find new equilibrium.



The change of equilibrium after investment

The case, when the governor spends money by himself (for his own pleasure), gives the same aggregate demand function, so the level of wages, industrial employment and small business employment will be the same, but the distribution of income will be different. The governor accumulates the significant part of the total regional income and spends it on his own purposes, so it can cause the discontent of voters.

4.2 Subsidies to Small Business

If the total additional revenues are equal to $t \cdot d\Pi(\alpha)$, then the subsidy for each firm is equal to $\frac{t \cdot d\Pi(\alpha)}{n}$. The equation (3) should be rewritten as:

$$\pi_i = \lambda (p_i - c) q_i - g_i + \frac{t \cdot d\Pi(\alpha)}{n} \to \max_{p_i, q_i}$$

Obviously, it does not change optimal p_i and q_i. The equation (7) becomes:

$$\lambda \frac{\mu \cdot I}{\sigma \cdot n} - \overline{g} + \frac{t \cdot d\Pi(\alpha)}{n} = rV_u \tag{26}$$

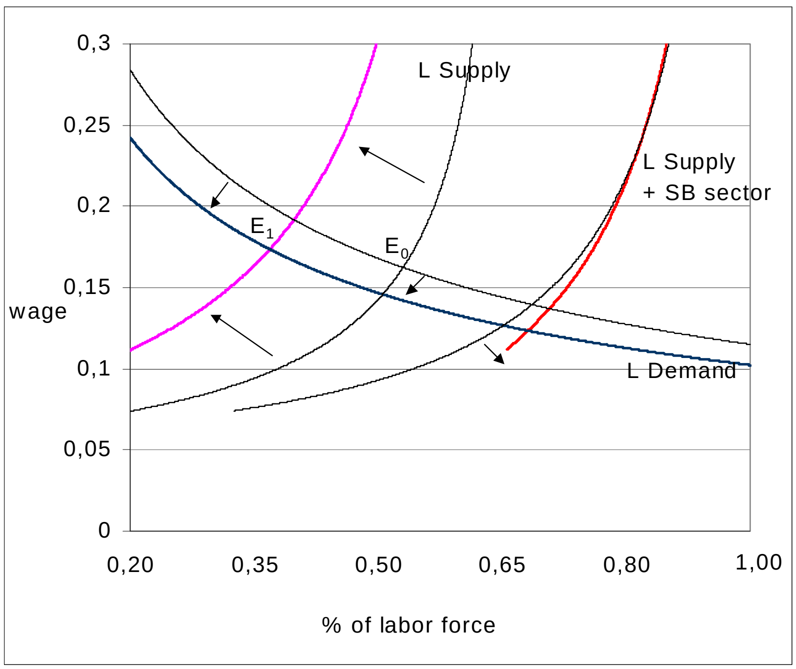
and the equation (8) changes to:

$$\overline{\pi} = \lambda \frac{\mu \cdot I}{\sigma \cdot n} - \frac{\overline{g}}{2} + \frac{t \cdot d\Pi(\alpha)}{n} \tag{27}$$

From the equations (6), (9) - (16) and (26) - (27) we can derive the new industrial labor supply curve:

$$L_{is}\left(w\right)=\frac{\overline{g}^{2}\left[\overline{g}\left(\sigma-\lambda\mu\right)+p_{b}\left(2\sigma-\lambda\mu\right)\right]-2p_{b}\left(\overline{g}+p_{b}\right)\sigma t\cdot d\Pi\left(L_{s}\left(w\right)\right)}{\left[\overline{g}\left(\sigma-\lambda\mu\right)+p_{b}\left(2\sigma-\lambda\mu\right)\right]\left(\overline{g}^{2}+2p_{b}p_{f}\cdot d\right)+2p_{b}\left(\overline{g}+p_{b}\right)\lambda\mu\cdot w} \tag{28}$$

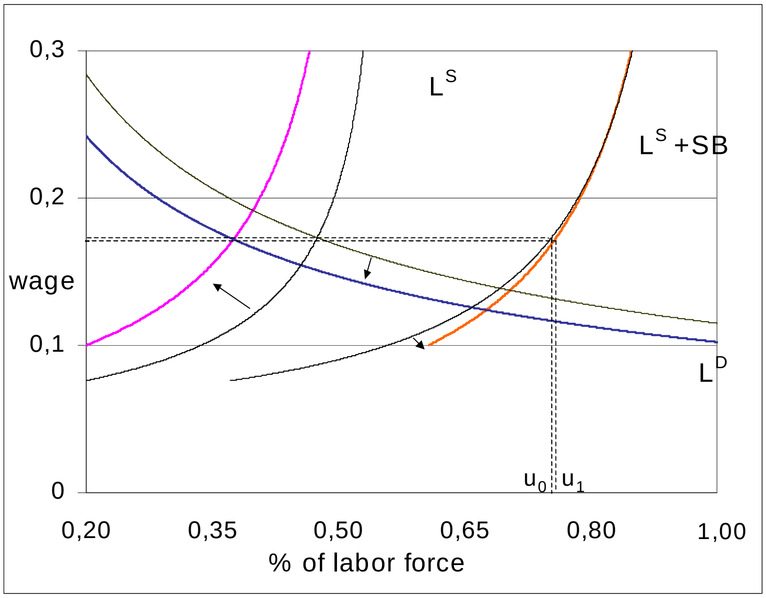
Using the new equation for labor supply and labor demand, we can find new equilibrium.



The change of equilibrium after giving subsidies to small business

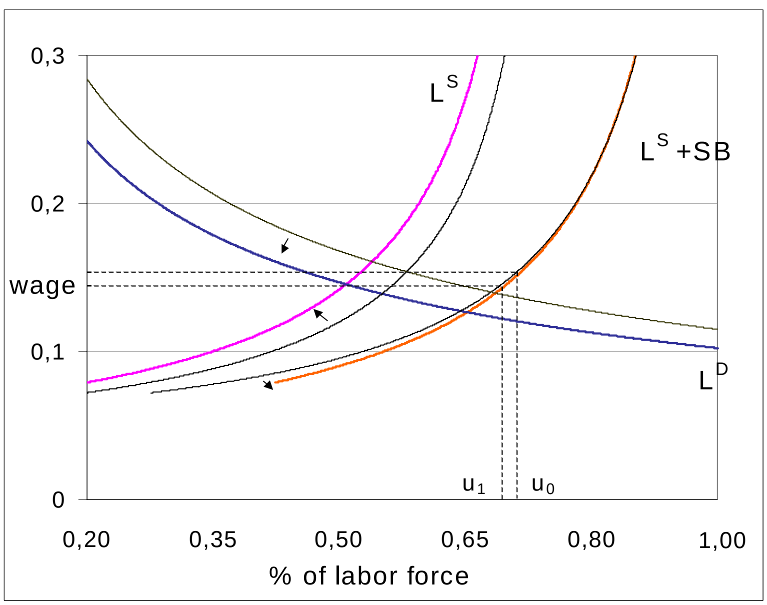
4.3 The Comparison of Regions with Different λs

There are two regions, the first has friendly small business environment ($\lambda = 0.8$) and the second has unfriendly small business environment ($\lambda = 0.4$). All other parameters are the same and $\widehat{w}$ is greater than equilibrium level of w. The investment leads to a decrease in labor demand, but since the first region has friendly small business environment, the additional taxes increase "total labor supply" more than they decrease in labor demand:



The impact of investment, friendly small business environment

The investment does not lead to an increase of unemployment, so the governor does not have any reasons to object to the restructuring. The situation is worse in the region with unfriendly small business environment:



The impact of investment, unfriendly small business environment

So, the investment leads to an increase in unemployment, and if the negotiations between the governor and the investor is impossible (or has high costs) and the governor has high θ, then the governor objects the restructuring.

But the region with friendly small business environment has higher level of wages, so the production costs higher. So, if the level of small business environment is too high then the investor also may not come.

4.4 Excess Employment

The governor can uphold excess employment in different ways. Informally, he can require not to fire workers in exchange for not increasing regulatory burden. Also, the governor can offer tax exemption if the investor increases employment. Formally, the governor may be able to increase firing costs – e.g. require the firm pay unemployed very high severance payments so that firing will never occur in equilibrium.

Let Π_1 be the profit of the investor in the new equilibrium (without hoarding labor). So, the outside option of the investor is equal to $(1-t)\,\Pi_1$. Let the governor have all the bargaining power. He can offer the investor to hire extra staff and give tax break for the investor. The governor also should collect taxes $t\Pi_0$, which he collected in the previous equilibrium, so the gross profit of the investor should be equal to $(1-t)\,\Pi_1 + t\Pi_0$. So we can find the new equilibrium using equation (17) for labor supply and budget constraint for the governor:

$$(1-t)\,\Pi_1 + t\Pi_0 = \left(K_1^a + L_{is}\,(w)^a\right)^{\frac{1}{a}\left(1-\frac{1}{\varepsilon}\right)} - wL_{is}\,(w)$$

Since the right side of this equation is a decreasing function of w and the left side is constant, this equation has a unique solution.

4.5 Regional Budget Revenues vs. Unemployment

Now it is time to analyze the trade-off between regional budget revenues and excess employment. The investor hires workers until the marginal productivity higher than the wage, but the governor can offer the investor a deal: the investor hires extra staff and the governor gives to the investor the tax break which is equal to the amount of the investor's losses from hiring extra staff. Let w_0 is the equilibrium level of wages and $\Pi\,(L_{is}(w_0), w_0) = \Pi_0$ is the equilibrium profit of the investor, then $(1-t)\,\Pi_0$ is the net profit of the investor and $t\Pi_0$ is the regional budget revenue. If the governor offers to increase the employment from $L_{is}\,(w_0)$ to $L_{is}\,(w_1)$ then the profit of the investors falls from Π_0 to $\Pi\,(L_{is}(w_1), w_1) = \Pi_1$. So the governor should offer the tax exemption, such as the investor's net profit remains the same (otherwise he refuses to hire extra staff), so the new regional budget revenue will be $\Pi_1 - (1-t)\,\Pi_0$, and the amount of the regional tax losses will be equal to $t\Pi_0 - [\Pi_1 - (1-t)\,\Pi_0] = \Pi_0 - \Pi_1$.

Since $u\,(w) = 1 - L_s(w)$, the maximization of the utility function (1) is equivalent to maximization

$$-\theta \cdot (1 - L_s(w_1)) + t\Pi_0 - \Pi_0 + \Pi(L_{is}(w_1), w_1) \to \max_{w_1}$$

which is equivalent to maximization of the following function:

$$\widetilde{U}(w) = \theta L_s(w) - \Pi(L_{is}(w), w) \to \max_{w}$$

For simplicity let r be equal to 0 for further analysis.

4.5.1 Taxes are spent outside the region

If the governor spends taxes outside the region (taxes are paid to governor who consumes imported goods and services) then the industrial labor supply function will be (17) and the total labor supply function $L_s(w)$ will be (21). Let's analyze the neighborhood of the equilibrium without excess employment. It follows from the envelope Proposition, that $d\Pi = -L_0 \cdot dw \Longrightarrow$

$$d\widetilde{U}(w) = \theta \cdot \frac{\partial L_s(w)}{\partial w} \cdot dw - L_0 \cdot dw$$

where $L_0 = L_{is}(w_0)$.

If $d\widetilde{U}(w) > 0$ then the governor requires excess employment.

Proposition 2.

If there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma)$ and the first region has better small business environment than the second has $(\lambda_1 > \lambda_2)$ then $\frac{\partial L_s(w,\lambda_1)}{\partial w} < \frac{\partial L_s(w,\lambda_2)}{\partial w}$ for all w, such as $p_f d + \varepsilon \le w \le \widehat{w}$. ε is chosen such as $u(p_f d + \varepsilon, \lambda) = 1 - L_s(p_f d + \varepsilon, \lambda) < 0.8$ for all λs. $\widehat{w} = \frac{2(p_f d)^2}{p_b} + p_f d$

Proof.

See Appendix 2.

It follows from *Proposition 2* that if two regions are not completely depressive (unemployment rate is less than 80%) then an additional unit of excess employment is more expensive in the region with higher level of the small business environment.

Since if unemployment rate is greater than 50% then people usually organize a revolution and change the governor (or kill him), the next outcome can be formulated: *the additional unit of excess employment is **always** more expensive in the region with higher small business environment.*

Proposition 3.

If there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma)$, the first region has better small business environment than the second has $(\lambda_1 > \lambda_2)$ and these two regions have the same equilibrium level of industrial employment $(L_{is}(w_1, \lambda_1) = L_{is}(w_2, \lambda_2))$ then $\frac{\partial \widetilde{U}(w_1,\lambda_1)}{\partial w} < \frac{\partial \widetilde{U}(w_2,\lambda_2)}{\partial w}$ for all θs and for all possible w_1, w_2, such as $p_f d+\varepsilon \leq w_1, w_2 \leq \widehat{w}$. ε is chosen such as $u(p_f d+\varepsilon, \lambda) = 1 - L_s\left(p_f d+\varepsilon, \lambda\right) < 0.8$ for all λs. $\widehat{w} = \frac{2(p_f d)^2}{p_b} + p_f d$

Proof.

See Appendix 2.

It follows from *Proposition 3*, that the governor of the region with low small business environment has more incentives to require excess employment than the governor of the region with high small business environment.

Corollary from *Proposition 3.*

For the set of parameters $p_b, p_f, d, \mu, \sigma, \theta$ and L (industrial employment), there is $\overline{\lambda} = \overline{\lambda}\left(p_b, p_f, d, \mu, \sigma, L, \theta\right)$,such as if $\lambda < \overline{\lambda}$ then the governor requires excess employment and if $\lambda > \overline{\lambda}$ then the governor does not require excess employment.

It is obvious that $\overline{\lambda}$ is a increasing function of θ (the higher the governor cares about the unemployment rate, the better small business environment should be, so that the governor does not require excess employment).

It is also easy to show that $\overline{\lambda}$is a decreasing function of L (it follows from the fact that $L_s\left(w\right)$ is a concave function and $L_{is}(w)$ is an increasing function). So if the region already have high level of industrial employment, the incentives for the governor to require excess employment are low.

4.5.2 Transfers to Households

This subsection analyze the case when the governor spends taxes on transfers to households. The same quantative results (aggregate demand, equilibrium level of wages, employment, etc.) will be received if taxes are misspent by governor and his cronies but within the region, or taxes are bribes that are also spent inside the region .

Let B is the size of the regional budget and the governor spends B on transfers to households. Using (23), the industrial labor supply function will be the following:

$$L_{is}\left(w\right) = \left(1 - \frac{p_f d}{w}\right)\left(1 - \frac{\lambda\mu\left(\overline{g}+p_b\right)}{\sigma\left(\overline{g}+2p_b\right)}\right) - \frac{\left(\overline{g}+p_b\right)\lambda\mu \cdot B}{\sigma\left(\overline{g}+2p_b\right)w}$$

Using (21) we can derive the equation for total labor supply:

$$L_s\left(w\right) = \frac{w - p_f d}{w} - \frac{\lambda\mu}{\sigma}\left(\frac{w - p_f d + B}{\overline{g}+2p_b}\right)\left[\frac{\left(\overline{g}+p_b\right)}{w} - \frac{\overline{g}}{w - p_f d}\right] \qquad (29)$$

Proposition 4.

If there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma)$, with the same size of regional budget $B > 0$, which the governors spend on transfers to households, and the first region has better small business environment than the second has $(\lambda_1 > \lambda_2)$ then $\frac{\partial L_s(w,\lambda_1)}{\partial w} < \frac{\partial L_s(w,\lambda_2)}{\partial w}$ for all w, such as $p_f d + \varepsilon \le w \le \widehat{w}$. ε is chosen such as $u(p_f d + \varepsilon, \lambda) = 1 - L_s(p_f d + \varepsilon, \lambda) < 0.65$ for all λs. $\widehat{w} = \frac{2(p_f d)^2}{p_b} + p_f d$

Proof.

See Appendix 2.

In contrast to *Proposition 2,* in case of *Proposition 4* there are two opposites effects (see (37)): the first effect $\frac{\lambda\mu}{\sigma} \cdot -\frac{1}{w(\overline{g}+2p_b)} \left[\frac{p_f d(\overline{g}+p_b)}{w} - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right]$ is the same as in case of the *Proposition 2,* and it is a decreasing function of λ when $u(w,\lambda) < 0.8$ and $w \le \widehat{w}$.The situation with the second effect, $\frac{\lambda\mu}{\sigma} \cdot -\frac{\partial B}{\partial w}\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right]$ or $\frac{\lambda\mu}{\sigma} \cdot L_{is}(w,\lambda)\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right]$ is not so obvious. Though $\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] < 0$, but $L_{is}(w,\lambda)$ is a decreasing function of λ, so the overall effect is not straightforward. It can be shown, that for some w the second effect is positive, but if w is quite high (such as $u(w,\lambda) < 0.65$) then the first effect is always dominated the second one and the overall effect of λ on $\frac{\partial L_s(w,\lambda)}{\partial w}$ is always negative.

The second effect is connected with the budget losses of the excess employment ($\frac{\partial B}{\partial w} = -L_{is}(w,\lambda)$). For small λ, the budget losses are higher (since $L_{is}(w,\lambda)$ is a decreasing function of λ), and the raise in wages leads not only to an increase of industrial employment, but also to a decrease of transfers to households, which causes a decrease of the demand for the production of small business sector, which decrease the employment in this sector of economy.

The third effect $\frac{\lambda\mu}{\sigma}B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right]$, which is connected with the initial size of the regional budget is always negative (when $u(w,\lambda) < 0.8$), so if B is quite high then the third effect dominates the second one and $\frac{\partial L_s(w,\lambda)}{\partial w}$is a decreasing function of λ for all w, such as $u(w,\lambda) < 0.8$.

Proposition 5.

If there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma)$, with the same size of the regional budget $B > 0$, which the governors spend on transfers to households, and the first region has better small business environment than the second has $(\lambda_1 > \lambda_2)$ and these two regions have the same equilibrium level of industrial employment $(L_{is}(w_1,\lambda_1) = L_{is}(w_2,\lambda_2))$ then $\frac{\partial \widetilde{U}(w_1,\lambda_1)}{\partial w} < \frac{\partial \widetilde{U}(w_2,\lambda_2)}{\partial w}$for all θs and for all possible w_1, w_2, such as $p_f d + \varepsilon \le w_1, w_2 \le \widehat{w}$. ε is chosen such as $u(p_f d + \varepsilon, \lambda) = 1 - L_s(p_f d + \varepsilon, \lambda) < 0.65$ for all λs. $\widehat{w} = \frac{2(p_f d)^2}{p_b} + p_f d$

Proof.

See Appendix 2.

It follows from *Proposition 5* that in case of transfers to households, the governor of the region with friendly small business environment is less inclined to require excess employment than the governor of the region with unfriendly small business environment.

Proposition 6.

If there are two regions with identical characteristics $(p_b, p_f, d, \lambda, \mu, \sigma)$, with the same size of the regional budget B, and the governor of the first region spends the budget outside the region whereas the governor of the second region spends the budget inside the region (or gives transfers to households) then $\frac{\partial L_{s1}(w)}{\partial w} > \frac{\partial L_{s2}(w)}{\partial w}$ for each $w < \widehat{w}$ ($L_{si}(w)$ – the total labor supply of the region i).

Proof.

See Appendix 2.

It follows from *Proposition 6* that the governor who spends money inside the region has less incentives to require excess employment than the governor who spends money outside the region. In case of spending money inside the region (or transfers to households), the excess employment leads to a fall of the regional budget revenues; as a result, it decreases the demand for production of the small business sector and leads to a decrease small business employment. So the total increase of employment will be less than in case of spending money outside the region.

4.6 Subsidies to Small Business vs. Transfers to Households

Assume that the governor collects T dollars of taxes and spends S dollars on subsidies to small business. So, the net budget of the region, is equal to $T - S$, which the governor spends on transfers to households (or spends money by himself inside the region).

The initial industrial labor supply function is the following (derived from (22), (28)):

$$L_{is}(w,S) = \frac{w - p_f d}{w}\left(1 - \frac{\lambda\mu\left(\overline{g}+p_b\right)}{\sigma\left(\overline{g}+2p_b\right)}\right) - \frac{\left(\overline{g}+p_b\right)\lambda\mu\cdot(T-S)}{\sigma\left(\overline{g}+2p_b\right)w} - \frac{\left(\overline{g}+p_b\right)S}{\left(\overline{g}+2p_b\right)w} \tag{30}$$

The initial total labor supply can be derived from (19), (30), and using the definition of $L_s(w)$ ($L_s(w) = L_{is}(w) + n_{sb}(w)$):

$$
\begin{aligned}
L_s(w,S) = & \frac{w-p_f d}{w} - \frac{\lambda\mu}{\sigma(\overline{g}+2p_b)}\left[\frac{(w-p_f d)(\overline{g}+p_b)}{w} - \overline{g}\right] - \\
& - \frac{\lambda\mu}{\sigma}\frac{T-S}{(\overline{g}+2p_b)}\left[\frac{(\overline{g}+p_b)}{w} - \frac{\overline{g}}{(w-p_f d)}\right] - \\
& - \frac{S}{(\overline{g}+2p_b)}\left[\frac{(\overline{g}+p_b)}{w} - \frac{\overline{g}}{(w-p_f d)}\right]
\end{aligned}
$$

The utility function of the governor is equal to:

$$U_{gov}(w,S) = (T-S) - \theta \cdot (1 - L_s(w,S))$$

If the governor decides to increase subsidies by dS, the total labor supply will be the following:

$$
\begin{aligned}
L_s(w,S+dS) = & \frac{w-p_f d}{w} - \frac{\lambda\mu}{\sigma(\overline{g}+2p_b)}\left[\frac{(w-p_f d)(\overline{g}+p_b)}{w} - \overline{g}\right] - \\
& - \frac{\lambda\mu}{\sigma}\frac{T-S-dS}{(\overline{g}+2p_b)}\left[\frac{(\overline{g}+p_b)}{w} - \frac{\overline{g}}{(w-p_f d)}\right] - \\
& - \frac{S+dS}{(\overline{g}+2p_b)}\left[\frac{(\overline{g}+p_b)}{w} - \frac{\overline{g}}{(w-p_f d)}\right]
\end{aligned}
$$

So, the difference $dL_s(w)$ is equal to:

$$dL_s(w) = dS\left(1 - \frac{\lambda\mu}{\sigma}\right)\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)} - \frac{(\overline{g}+p_b)}{w}\right]$$

The derivative of the governor's utility function is equal to:

$$\frac{\partial U_{gov}(w,S)}{\partial S} = -1 + \theta\left(1 - \frac{\lambda\mu}{\sigma}\right)\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)} - \frac{(\overline{g}+p_b)}{w}\right] \tag{31}$$

If $\frac{\partial U_{gov}(w,S)}{\partial S} > 0$ then the governor decides to increase subsidies, otherwise he prefers to cut down the subsidies.

Proposition 7.

If there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma)$, and the first region has better small business environment than the second has $(\lambda_1 > \lambda_2)$ then $\frac{\partial U_{gov}(w,S,\lambda_1)}{\partial S} < \frac{\partial U_{gov}(w,S,\lambda_2)}{\partial S}$ for all w, such as $w < \widehat{w}$.

Proof.

See Appendix 2.

In other words, *Proposition 7* says that the governor of the region with high level of small business environment has less incentives to give subsidies to small business than the governor of the region with low level of small business environment. It is easy to show that for the set of parameters $(p_b, p_f, d, \mu, \sigma, w)$ there is $\widetilde{\lambda} = \widetilde{\lambda}(p_b, p_f, d, \mu, \sigma, w)$, such as if $\lambda < \widetilde{\lambda}$ then the governor prefers to give subsidies to small business, otherwise he prefers to give transfers to households.

Proposition 8.

If there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma, \lambda)$, and the level of wages in the first region higher than the level of wages in the second region $(w_1 > w_2)$ then $\frac{\partial U_{gov}(w_1,S)}{\partial S} < \frac{\partial U_{gov}(w_2,S)}{\partial S}$ for all w_1, w_2, such as $w_1, w_2 < \widehat{w}$.

Proof.

See Appendix 2.

It follows from *Proposition 8,* that the governor of the region with high level of wages has less incentives to give subsidies to small business than the governor of the region with low level of wages.

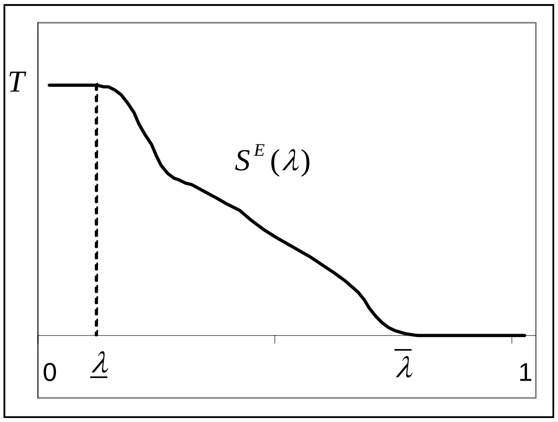
Proposition 9.

Let $L_D(w)$ be the labor demand function, such as $L_D(w)$ is a decreasing function of w and $L_D(\widehat{w}) = 0$ (this restriction is needed to guarantee that the equilibrium level of wages is less than $\widehat{w}$); the governor's utility function is $U_{gov}(w, S) = (T - S) - \theta \cdot (1 - L_s(w, S))$, where $L_s(w, S)$ is the total labor supply function; T is the amount of regional taxes; the governor spends S on subsidies to small business and $(T - S)$ on transfers to households (or spends by himself inside the region); T is fixed and S is chosen by the governor, such as $0 \le S \le T$. Then the equilibrium level of subsidies, S^E, is negatively depends on the level of small business environment, λ, in other words, if there are two regions with identical characteristics $(p_b, p_f, d, \mu, \sigma)$, but the first region has better small business environment than the second has $(\lambda_1 > \lambda_2)$ then $S^E(\lambda_1) \le S^E(\lambda_2)$.

Proof.

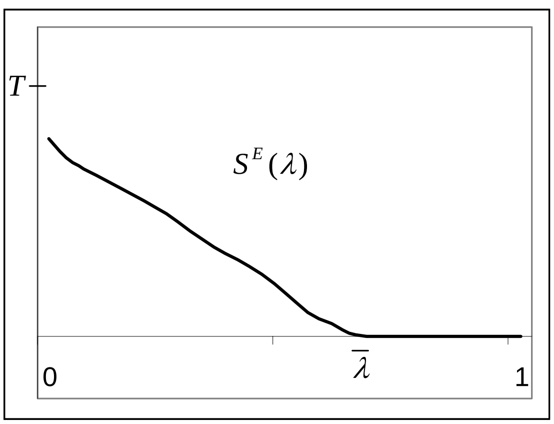
See Appendix 2.

There are four possible profiles of the equilibrium level of subsidies:



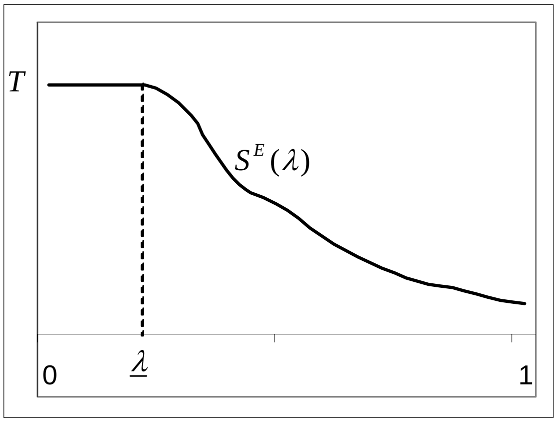
If $\lambda \leq \underline{\lambda}$ then the governor spends all taxes on subsidies, if $\underline{\lambda} < \lambda < \overline{\lambda}$ then the governor spends $S^E(\lambda)$ on subsidies and the rest of taxes $(T - S^E(\lambda))$ on transfers to households. If $\lambda \geq \overline{\lambda}$ then the subsidies are equal to 0 and the governor spends all taxes on transfers to households.

The second possible profile is the following:



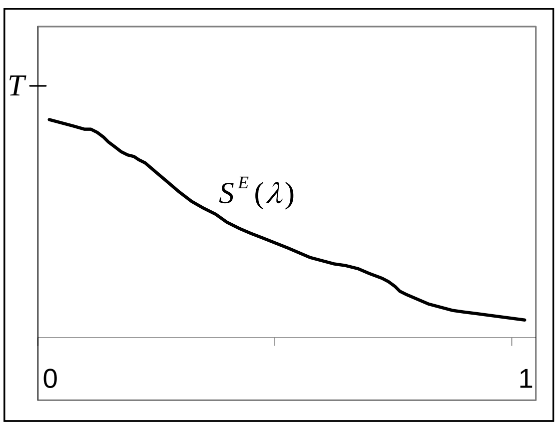
If $\lambda < \overline{\lambda}$ then the governor spends $S^E(\lambda)$ on subsidies and the rest of taxes $(T - S^E(\lambda))$ on transfers to households. If $\lambda \geq \overline{\lambda}$ then the subsidies are equal to 0 and the governor spends all taxes on transfers to households.

The third profile is



If $\lambda \leq \underline{\lambda}$ then the governor spends all taxes on subsidies, if $\underline{\lambda} < \lambda$ then the governor spends $S^E(\lambda)$ on subsidies and the rest of taxes $(T - S^E(\lambda))$ on transfers to households.

Finally, the last possible profiles of $S^E(\lambda)$ is the following:



For all possible λs, the governor spends $S^E(\lambda)$ on subsidies and the rest of taxes $(T - S^E(\lambda))$ on transfers to households.

Also, it is easy to show that the higher θ is, the higher the equilibrium level of subsidies is. It is a quite trivial fact, so there is no any interest to discuss it more.

5 Conclusion

Though this model is very simple, it incorporates two opposite effects of small business environment on the investment climate. On one hand, friendly small business environment decreases unemployment, so the governor does not have incentives to object investment and restructuring. On the other hand, the high level of small business environment increases wages in the regional labor market, and therefore reduces investors' profits.

We show that the level of small business environment affects the impact of different regional economic policies such as profit taxes, subsidies to small businesses, (formal or informal) regulation of firing costs. We show that better small business environment decreases governor's incentives to uphold excess employment in large firms, and to give subsidies to small businesses. So, the governor of the region with high level of small business environment interferes in the economic processes less than the governor of the region with low level of small business environment. Therefore the economy of the region with high level of small business environment is less distorted. The fiscal spending of the region with high level of small business environment is also better: the governor prefers transfers to households than to give subsidies to small firms. The direct subsidies to business often lead to corruption, and therefore further distortions in the economy.

The governor of the region with friendly small business environment does not object investment and restructuring, because almost all redundant workers can open small firms, whereas the governor of the region with unfriendly small business environment resists to restructuring because of the fear of unemployment; even if he allows the restructuring. But as it was shown in this paper, the region with high level of small business environment has higher level of wages than the region with low level of small business environment, so the production costs in the region with high level of small business environment could be too high, and the investor may not come (even if the governor does not object restructuring).

The model can be improved a lot, if the small business environment becomes endogenous rather that exogenous. There are several possible ways how to endogenize it. Firstly, we can assume that there is some function $\lambda(I_\lambda)$, there I_λ is the investment in small business environment and the governor can enhance the small business environment by investing a part of the regional budget in it. Secondly, the governor can weaken the regulation burden, if he reforms the system of licensing and control of small business – e.g. he can increase the financing of the police, firemen, sanitary service, etc. in exchange for not to blackmail small business. Finally, the governor can release new-born firms from inspections for some period of time. There could be other ways how the governor can influence on small business environment. Including endogenous small business environment in the model, we can also enlarge the area of possible negotiations between the governor and the investor: the investor can invest money in small business environment in exchange for the governor's approval of restructuring.

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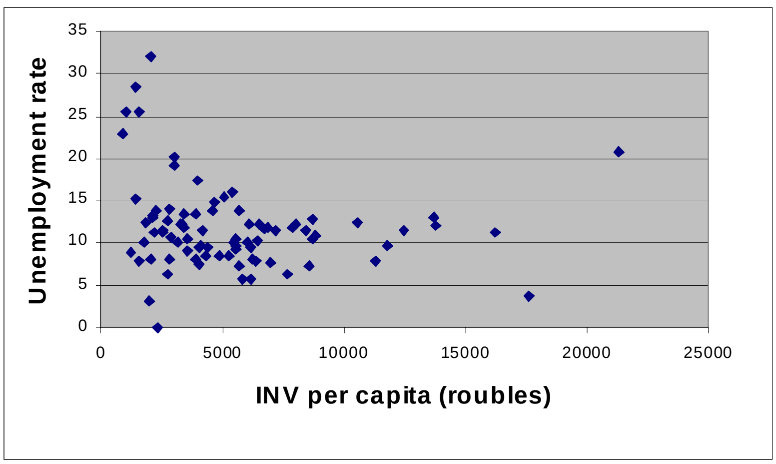
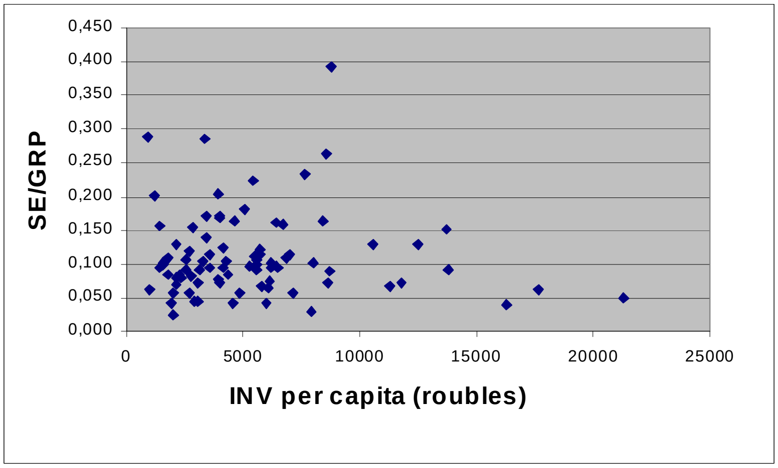
A Appendix 1

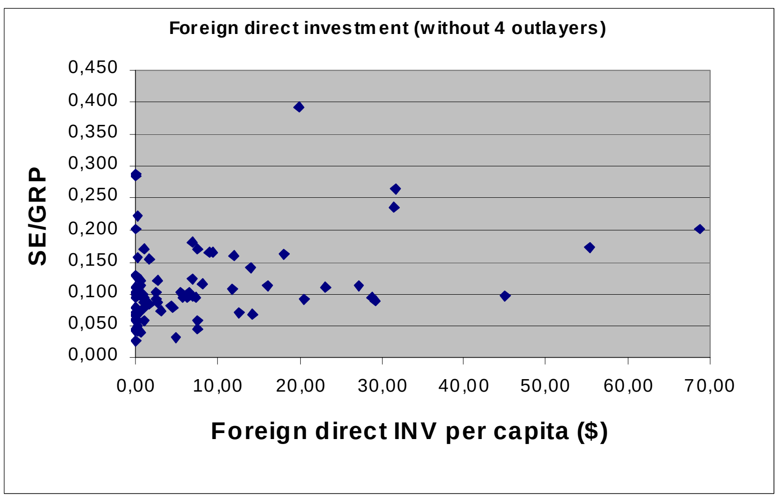
The data (2000 year) include all Russian regions, excluding Tyumen oblast, Khanty-Mansi autonomous okrug, Yamalo-Nenets autonomous, and Chechnya republic. SE/GRP – the share of small enterprises in gross regional product, INV/GRP – the investment in fixed capital to GRP, INV per capita - the investment in fixed capital per capita. The source of data is “Goskomstat”, the official Russia’s statistical agency.



Unemployment rate and investment per capita



Share of small business in GRP and investment per capita



Share of small business in GRP and foreign direct investment per capita

B Appendix 2

B.1 Labor Supply Curve

For simplicity, let b be equal to 0. From (11) and (13) we can receive

$$rV_u = p_e \cdot d \tag{32}$$

From (12) and (13):

$$rV_u = w - (p_f + r)\, d \tag{33}$$

Using (7) and (8):

$$\overline{\pi} = rV_u + \frac{\overline{g}}{2} \tag{34}$$

From (9) and (34) we can derive:

$$(V_s - V_u)(r + p_b) = \frac{\overline{g}}{2}$$

And using (10):

$$rV_u = \frac{\overline{g}^2}{2(r + p_b)} \tag{35}$$

From (32) - (35) and using (6) and (16) we can derive the system of equations:

$$\begin{aligned} w - (p_f + r)\, d &= \frac{\overline{g}^2}{2(r + p_b)} \\ w - (p_f + r)\, d &= \overline{g}\left(\frac{\lambda\mu}{2\sigma} - 1\right) + \frac{\lambda\mu \cdot \alpha w}{\sigma n} \\ w - (p_f + r)\, d &= \frac{\alpha \cdot p_f \cdot d}{1 - \alpha - n - \frac{p_b \cdot n}{\overline{g}}} \end{aligned}$$

From this system we can find $\overline{g}(w)$:

$$\overline{g}(w) = \sqrt{2(r + p_b)(w - (p_f + r)\, d)}$$

And we can find $n_{sb}(w, \alpha)$ – the employment in small business:

$$n_{sb}(w, \alpha) = \frac{\overline{g}(w)}{\overline{g}(w) + p_b} - \alpha\left(\frac{\overline{g}(w)}{\overline{g}(w) + p_b}\right)\left(1 + \frac{p_f \cdot d}{w - (p_f + r)\, d}\right)$$

Finally, we can derive $\alpha(w)$:

$$\alpha(w) = \frac{\overline{g}^2(w)\left[\overline{g}(w)(\sigma - \lambda\mu) + p_b(2\sigma - \lambda\mu)\right]}{\left[\overline{g}(w)(\sigma - \lambda\mu) + p_b(2\sigma - \lambda\mu)\right]\left(\overline{g}^2(w) + 2p_b p_f \cdot d\right) + 2p_b\left(\overline{g}(w) + p_b\right)\lambda\mu \cdot w}$$

B.2 Proof of Proposition 1

Firstly, it can be shown that the industrial labor supply function (17) is a decreasing function of λ.(17) can be rewritten as:

$$L_{is}(w,\lambda) = \frac{\overline{g}^2}{\left(\overline{g}^2 + 2p_b p_f \cdot d\right) + \frac{2p_b(\overline{g}+p_b)\lambda\mu\cdot w}{[\overline{g}(\sigma-\lambda\mu)+p_b(2\sigma-\lambda\mu)]}}$$

Obviously, $\frac{2p_b(\overline{g}+p_b)\lambda\mu\cdot w}{[\overline{g}(\sigma-\lambda\mu)+p_b(2\sigma-\lambda\mu)]}$ is an increasing function of λ, thus $L_{is}(w,\lambda)$ is a decreasing function of λ. After that, let's find the solution of the equation

$$1 - \left(\frac{\overline{g}(\widehat{w})}{\overline{g}(\widehat{w}) + p_b}\right)\left(1 + \frac{p_f \cdot d}{\widehat{w} - (p_f + r)\, d}\right) = 0$$

Using fact that $\overline{g}^2(w) = 2(r + p_b)(w - (p_f + r)\, d)$, we can obtain the following equation:

$$1 + \frac{2(r + p_b)\, p_f \cdot d}{\overline{g}^2(\widehat{w})} = \frac{\overline{g}(\widehat{w}) + p_b}{\overline{g}(\widehat{w})}$$

Multiplying both sides by $\overline{g}^2(\widehat{w})$ we receive the unique solution of this equation

$$g(\widehat{w}) = 2(r + p_b)\frac{p_f \cdot d}{p_b}$$

and

$$\widehat{w} = 2(r + p_b)\frac{(p_f d)^2}{p_b^2} + (p_f + r)\, d$$

Since $1 - \left(\frac{\overline{g}(w)}{\overline{g}(w)+p_b}\right)\left(1 + \frac{p_f \cdot d}{w-(p_f+r)d}\right)$ is a continuous function for $w > (p_f + r)\, d$ and

$$1 - \left(\frac{\overline{g}((p_f + r)\, d + \varepsilon)}{\overline{g}((p_f + r)\, d + \varepsilon) + p_b}\right) \cdot \left(1 + \frac{p_f \cdot d}{\varepsilon}\right) \to -\infty \text{ when } \varepsilon \to 0$$

then $1 - \left(\frac{\overline{g}(w)}{\overline{g}(w)+p_b}\right) \cdot \left(1 + \frac{p_f \cdot d}{w-(p_f+r)d}\right) < 0$ if $w < \widehat{w}$.

Since $\left[1 - \left(\frac{\overline{g}(\widehat{w})}{\overline{g}(\widehat{w})+p_b}\right)\left(1 + \frac{p_f \cdot d}{\widehat{w}-(p_f+r)d}\right)\right]' \neq 0$ then

$$1 - \left(\frac{\overline{g}(w)}{\overline{g}(w) + p_b}\right)\left(1 + \frac{p_f \cdot d}{w - (p_f + r)\, d}\right) > 0 \text{ if } w > \widehat{w}$$

Since $L_{is}(w,\lambda)$ is a decreasing function of λ and $1-\left(\frac{\overline{g}(w)}{\overline{g}(w)+p_b}\right)\left(1 + \frac{p_f \cdot d}{w-(p_f+r)d}\right)$ does not depend on λ, we can obtain that if $w < \widehat{w}$ then $L_s(w,\lambda)$ is an increasing function of λ, if $w > \widehat{w}$ then $L_s(w,\lambda)$ is a decreasing function of λ, and if $w = \widehat{w}$ then $L_s(w,\lambda)$ does not depend on λ.

QED.

B.3 Proof of Proposition 2

Firstly, let's find the derivative $\frac{\partial \overline{g}(w)}{\partial w}$:

$$\frac{\partial \overline{g}\left(w\right)}{\partial w}=\frac{\partial\sqrt{2p_b\left(w-p_fd\right)}}{\partial w}=\sqrt{\frac{p_b}{2\left(w-p_fd\right)}}=\frac{p_b}{\overline{g}}$$

Substituting (20) in (21) we can calculate $L_s(w)$:

$$L_s\left(w\right)=\frac{\overline{g}}{\overline{g}+p_b}+\left(1-\frac{p_fd}{w}\right)\left(1-\frac{\lambda\mu\left(\overline{g}+p_b\right)}{\sigma\left(\overline{g}+2p_b\right)}\right)\left[1-\left(\frac{\overline{g}}{\overline{g}+p_b}\right)\left(1+\frac{p_fd}{w-p_fd}\right)\right]$$

or

$$L_s\left(w\right)=\frac{w-p_fd}{w}-\frac{\lambda\mu}{\sigma}\left[\frac{w-p_fd}{w}\frac{\left(\overline{g}+p_b\right)}{\left(\overline{g}+2p_b\right)}-\frac{\overline{g}}{\left(\overline{g}+2p_b\right)}\right]$$

Now we can find the derivative $\frac{\partial L_s(w)}{\partial w}$:

$$\begin{aligned}\frac{\partial L_s\left(w\right)}{\partial w} &= \frac{p_fd}{w^2}-\frac{\lambda\mu}{\sigma}\frac{1}{w\left(\overline{g}+2p_b\right)}\left[\frac{p_fd\left(\overline{g}+p_b\right)}{w}-\frac{2p_b^2w}{\left(\overline{g}+2p_b\right)\overline{g}}+\frac{p_b\overline{g}}{2\left(\overline{g}+2p_b\right)}\right]= \\ &= \frac{p_fd}{w^2}-\frac{\lambda\mu}{\sigma}\frac{1}{w\left(\overline{g}+2p_b\right)}\left[\frac{p_fd\left(\overline{g}+p_b\right)}{w}-\frac{p_b^2\left(w+p_fd\right)}{\left(\overline{g}+2p_b\right)\overline{g}}\right] \qquad (36)\end{aligned}$$

If $\left[\frac{p_fd(\overline{g}+p_b)}{w}-\frac{p_b^2(w+p_fd)}{(\overline{g}+2p_b)\overline{g}}\right]>0$ then the greater λ is, the lower $\frac{\partial L_s(w)}{\partial w}$ and Proposition will be proved.

If $\lambda=0$ then $L_s\left(w\right)=\frac{w-p_fd}{w}$, since $L_s\left(w,\lambda\right)$ is an increasing function of λ (see *Proposition 1*), if $w\geq 1.25p_fd$ then $1-L_s\left(w,\lambda\right)=u\left(w,\lambda\right)\leq 0.8$ for all λs.

Using 10^8 Monte Carlo simulations it was proven that $\left[\frac{p_fd(\overline{g}+p_b)}{w}-\frac{p_b^2(w+p_fd)}{(\overline{g}+2p_b)\overline{g}}\right]>0$ for $p_b\in(0,1)$, $p_fd\in(0,1)$ and $w\in\left(1.25p_fd,\frac{2(p_fd)^2}{p_b}+p_fd\right)$. All used distributions of p_fd, p_b, w were uniformed.

So if $(\lambda_1>\lambda_2)$ then $\frac{\partial L_s(w,\lambda_1)}{\partial w}<\frac{\partial L_s(w,\lambda_2)}{\partial w}$ for all w, such as $1.25p_fd\leq w\leq\frac{2(p_fd)^2}{p_b}+p_fd$.

QED.

B.4 Proof of Proposition 3

Since $L_{is}\left(w,\lambda\right)$ is a decreasing function of λ, if $\lambda_1>\lambda_2$ and $L_{is}\left(w_1,\lambda_1\right)=L_{is}\left(w_2,\lambda_2\right)$ then $w_1>w_2$. Since $L_s\left(w\right)$ is a concave function, $\frac{\partial L_s(w)}{\partial w}$is a decreasing function of w. Using the result of *Proposition 2*, it follows that

$$\frac{\partial L_s\left(w_1,\lambda_1\right)}{\partial w}<\frac{\partial L_s\left(w_2,\lambda_1\right)}{\partial w}<\frac{\partial L_s\left(w_2,\lambda_2\right)}{\partial w}$$

for all w_1, w_2, such as $1.25 p_f d \le w_1, w_2 \le \frac{2(p_f d)^2}{p_b} + p_f d$.

Using the fact that $\frac{\partial \widetilde{U}(w)}{\partial w} = \theta \cdot \frac{\partial L_s(w)}{\partial w} - L$, it follows that

$$\begin{aligned} \frac{\partial \widetilde{U}(w_1,\lambda_1)}{\partial w} &= \theta \cdot \frac{\partial L_s(w_1,\lambda_1)}{\partial w} - L_{is}(w_1,\lambda_1) < \theta \cdot \frac{\partial L_s(w_2,\lambda_2)}{\partial w} - L_{is}(w_1,\lambda_1) = \\ &= \theta \cdot \frac{\partial L_s(w_2,\lambda_2)}{\partial w} - L_{is}(w_2,\lambda_2) = \frac{\partial \widetilde{U}(w_2,\lambda_2)}{\partial w} \end{aligned}$$

So, $\frac{\partial \widetilde{U}(w_1,\lambda_1)}{\partial w} < \frac{\partial \widetilde{U}(w_2,\lambda_2)}{\partial w}$.

QED.

B.5 Proof of Proposition 4

The formula (29) can be rewritten as

$$\begin{aligned} L_s(w) &= \frac{w - p_f d}{w} - \frac{\lambda\mu}{\sigma}\left[\frac{w - p_f d}{w}\frac{(\overline{g}+p_b)}{(\overline{g}+2p_b)} - \frac{\overline{g}}{(\overline{g}+2p_b)}\right] - \\ &\quad -\frac{\lambda\mu}{\sigma}B\left[\frac{(\overline{g}+p_b)}{w(\overline{g}+2p_b)} - \frac{\overline{g}}{(w-p_f d)(\overline{g}+2p_b)}\right] \\ &= \frac{w - p_f d}{w} - \frac{\lambda\mu}{\sigma}\left[\frac{w - p_f d}{w}\frac{(\overline{g}+p_b)}{(\overline{g}+2p_b)} - \frac{\overline{g}}{(\overline{g}+2p_b)}\right] - \\ &\quad -\frac{\lambda\mu}{\sigma}B\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] \end{aligned}$$

Now we can find the derivative $\frac{\partial L_s(w)}{\partial w}$:

$$\begin{aligned} \frac{\partial L_s(w)}{\partial w} &= \frac{p_f d}{w^2} - \frac{\lambda\mu}{\sigma}\frac{1}{w(\overline{g}+2p_b)}\left[\frac{p_f d(\overline{g}+p_b)}{w} - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] - \\ &\quad -\frac{\lambda\mu}{\sigma}\frac{\partial B}{\partial w}\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] - \\ &\quad -\frac{\lambda\mu}{\sigma}B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] \end{aligned} \tag{37}$$

Since $\frac{\partial B}{\partial w} = \frac{\partial \Pi}{\partial w}$, $\frac{\partial B}{\partial w} = -L_{is}(w) = -\left(1 - \frac{p_f d}{w}\right)\left(1 - \frac{\lambda\mu(\overline{g}+p_b)}{\sigma(\overline{g}+2p_b)}\right) + \frac{(\overline{g}+p_b)\lambda\mu\cdot B}{\sigma(\overline{g}+2p_b)w}$.
Then $\frac{\partial L_s(w)}{\partial w}$ is equal:

$$
\begin{aligned}
\frac{\partial L_s(w)}{\partial w} &= \frac{p_f d}{w^2} - \frac{\lambda\mu}{\sigma}\frac{1}{w(\overline{g}+2p_b)}\left[\frac{p_f d(\overline{g}+p_b)}{w} - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] - \\
&+\frac{\lambda\mu}{\sigma}\left[\left(1-\frac{p_f d}{w}\right)\left(1-\frac{\lambda\mu(\overline{g}+p_b)}{\sigma(\overline{g}+2p_b)}\right) - \frac{(\overline{g}+p_b)\lambda\mu\cdot B}{\sigma(\overline{g}+2p_b)w}\right]\cdot \\
&\cdot\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] - \\
&-\frac{\lambda\mu}{\sigma}B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2}+\frac{p_b}{w\overline{g}(\overline{g}+2p_b)}+\frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] \\
&= \frac{p_f d}{w^2} - \frac{\lambda\mu}{\sigma}\frac{1}{w(\overline{g}+2p_b)}\left[\frac{p_f d(\overline{g}+p_b)}{w} - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] - \\
&+\frac{\lambda\mu}{\sigma}\left(\frac{w-p_f d}{w}\right)\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] - \\
&-\left(\frac{\lambda\mu}{\sigma}\right)^2\frac{w-p_f d}{w}\frac{\overline{g}+p_b}{\overline{g}+2p_b}\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] - \\
&-\frac{\lambda\mu}{\sigma}B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2}+\frac{p_b}{w\overline{g}(\overline{g}+2p_b)}+\frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] - \\
&-\left(\frac{\lambda\mu}{\sigma}\right)^2\frac{(\overline{g}+p_b)B}{(\overline{g}+2p_b)w}\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] \\
&= \frac{p_f d}{w^2} - \frac{\lambda\mu}{\sigma}\frac{1}{w(\overline{g}+2p_b)}\left[\frac{2p_f d(\overline{g}+p_b)}{w} - p_b - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] - \\
&-\left(\frac{\lambda\mu}{\sigma}\right)^2\frac{\overline{g}+p_b}{(\overline{g}+2p_b)^2 w}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right] - \\
&-\frac{\lambda\mu}{\sigma}B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2}+\frac{p_b}{w\overline{g}(\overline{g}+2p_b)}+\frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] - \\
&-\left(\frac{\lambda\mu}{\sigma}\right)^2\frac{2p_b(\overline{g}+p_b)B}{(\overline{g}+2p_b)^2\overline{g}^2 w}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right]
\end{aligned}
$$

Now, we can calculate the derivative $\frac{\partial^2 L_s(w)}{\partial w\partial\lambda}$, and if $\frac{\partial^2 L_s(w)}{\partial w\partial\lambda} < 0$ then the Proposition will be proven.

$$
\begin{aligned}
\frac{\partial^2 L_s(w)}{\partial w\partial\lambda}\frac{\sigma}{\mu} &= -\frac{1}{w(\overline{g}+2p_b)}\left[\frac{2p_f d(\overline{g}+p_b)}{w} - p_b - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] - \\
&-\frac{2\lambda\mu}{\sigma}\frac{\overline{g}+p_b}{(\overline{g}+2p_b)^2 w}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right] - \\
&-B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2}+\frac{p_b}{w\overline{g}(\overline{g}+2p_b)}+\frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] - \\
&-B\frac{2\lambda\mu}{\sigma}\frac{2p_b(\overline{g}+p_b)B}{(\overline{g}+2p_b)^2\overline{g}^2 w}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right]
\end{aligned}
$$

or

$$\frac{\partial^2 L_s(w)}{\partial w \partial \lambda}\frac{\sigma}{\mu} = -C1(\lambda) - C2(\lambda) \cdot B$$

where

$$\begin{aligned} C1(\lambda) &= \frac{1}{w(\overline{g}+2p_b)}\left[\frac{2p_f d(\overline{g}+p_b)}{w} - p_b - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] + \\ &+ \frac{2\lambda\mu}{\sigma}\frac{\overline{g}+p_b}{(\overline{g}+2p_b)^2 w}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right] \end{aligned}$$

$$\begin{aligned} C2(\lambda) &= \left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] + \\ &+ \frac{2\lambda\mu}{\sigma}\frac{2p_b(\overline{g}+p_b)}{(\overline{g}+2p_b)^2\overline{g}^2 w}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right] \end{aligned}$$

Using Monte Carlo simulations ($p_b \in (0,1), p_f d \in (0,1)$ and $w \in \left(1.25 p_f d, \frac{2(p_f d)^2}{p_b} + p_f d\right)$), it can be shown that $\left[\frac{2p_f d(\overline{g}+p_b)}{w} - p_b - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] > 0$. If $p_b - \frac{(\overline{g}+p_b)p_f d}{w} > 0$ then the first component ($C1(\lambda)$) is greater than 0.So, we should analyze only the case when $p_b - \frac{(\overline{g}+p_b)p_f d}{w} < 0$.Using the fact, that $\frac{\lambda\mu}{\sigma} < 1$ for all possible λ, μ, σ it can be shown, that $C1(\lambda)$

$$\begin{aligned} &\sim \left[\frac{2p_f d(\overline{g}+p_b)}{w} - p_b - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] + \frac{2\lambda\mu}{\sigma}\frac{\overline{g}+p_b}{(\overline{g}+2p_b)}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right] > \\ &> \left[\frac{2p_f d(\overline{g}+p_b)}{w} - p_b - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] + 2\frac{\overline{g}+p_b}{(\overline{g}+2p_b)}\left[p_b - \frac{(\overline{g}+p_b)p_f d}{w}\right] = \overline{C1} \end{aligned}$$

Using Monte Carlo simulations ($p_b \in (0,1), p_f d \in (0,1)$ and $w \in \left(1.53 p_f d, \frac{2(p_f d)^2}{p_b} + p_f d\right)$), it can be shown that $\overline{C1} > 0$ (if $u(w,\lambda) \leq u(w,0) = 1 - \frac{w - p_f d}{w} < 0.65$ then $w > \frac{p_f d}{0.65} \approx 1.538 p_f d$).

So, we show, that $C1(\lambda) > 0$ for $w \in \left(1.53 p_f d, \frac{2(p_f d)^2}{p_b} + p_f d\right)$.

Analogically, using Monte Carlo simulations it can be shown, that $C2(\lambda) > 0$ for $w \in \left(1.25 p_f d, \frac{2(p_f d)^2}{p_b} + p_f d\right)$, for all possible λ, μ, σ.. Since $B \geq 0$, $\frac{\partial^2 L_s(w)}{\partial w \partial \lambda} < 0$ for $w \in \left(1.53 p_f d, \frac{2(p_f d)^2}{p_b} + p_f d\right)$.

QED.

B.6 Proof of Proposition 5

Since $L_{is}(w,\lambda)$ is a decreasing function of λ, if $\lambda_1 > \lambda_2$ and $L_{is}(w_1,\lambda_1) = L_{is}(w_2,\lambda_2)$ then $w_1 > w_2$.It can be shown that $\frac{\partial L_s(w)}{\partial w}$(37) is a decreasing function of w for all possible parameters $(p_b, p_f, d, \lambda, \mu, \sigma, B)$ and for all w. Using the result of *Proposition 4*, it follows that

$$\frac{\partial L_s(w_1,\lambda_1)}{\partial w} < \frac{\partial L_s(w_2,\lambda_1)}{\partial w} < \frac{\partial L_s(w_2,\lambda_2)}{\partial w}$$

for all w_1, w_2, such as $1.53 p_f d \leq w_1, w_2 \leq \frac{2(p_f d)^2}{p_b} + p_f d$.

Using the fact that $\frac{\partial \widetilde{U}(w)}{\partial w} = \theta \cdot \frac{\partial L_s(w)}{\partial w} - L$, it follows that

$$\begin{aligned}
\frac{\partial \widetilde{U}(w_1,\lambda_1)}{\partial w} &= \theta \cdot \frac{\partial L_s(w_1,\lambda_1)}{\partial w} - L_{is}(w_1,\lambda_1) < \theta \cdot \frac{\partial L_s(w_2,\lambda_2)}{\partial w} - L_{is}(w_1,\lambda_1) = \\
&= \theta \cdot \frac{\partial L_s(w_2,\lambda_2)}{\partial w} - L_{is}(w_2,\lambda_2) = \frac{\partial \widetilde{U}(w_2,\lambda_2)}{\partial w}
\end{aligned}$$

So, $\frac{\partial \widetilde{U}(w_1,\lambda_1)}{\partial w} < \frac{\partial \widetilde{U}(w_2,\lambda_2)}{\partial w}$.

QED.

B.7 Proof of Proposition 6

$\frac{\partial L_{s1}(w)}{\partial w}$ is equal to (36):

$$\frac{\partial L_{s1}(w)}{\partial w} = \frac{p_f d}{w^2} - \frac{\lambda\mu}{\sigma}\frac{1}{w(\overline{g}+2p_b)}\left[\frac{p_f d(\overline{g}+p_b)}{w} - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right]$$

$\frac{\partial L_{s2}(w)}{\partial w}$ can be derived from (37), using the fact that $\frac{\partial B}{\partial w} = -\,L_{is}(w)$:

$$\begin{aligned}
\frac{\partial L_{s2}(w)}{\partial w} &= \frac{p_f d}{w^2} - \frac{\lambda\mu}{\sigma}\frac{1}{w(\overline{g}+2p_b)}\left[\frac{p_f d(\overline{g}+p_b)}{w} - \frac{p_b^2(w+p_f d)}{(\overline{g}+2p_b)\overline{g}}\right] + \\
&+ \frac{\lambda\mu}{\sigma} L_{s2}(w)\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] - \\
&- \frac{\lambda\mu}{\sigma} B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right]
\end{aligned}$$

The difference $\frac{\partial L_{s1}(w)}{\partial w} - \frac{\partial L_{s2}(w)}{\partial w}$ is equal to:

$$\begin{aligned}
&\frac{\lambda\mu}{\sigma} B\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] - \\
&- \frac{\lambda\mu}{\sigma} L_{is2}(w)\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right]
\end{aligned}$$

The Proposition states that $\frac{\partial L_{s1}(w)}{\partial w} - \frac{\partial L_{s2}(w)}{\partial w} > 0$ for $w < \widehat{w}$, so if we prove that $\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] > 0$ and $\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] < 0$ then the Proposition will be proven.

Firstly, let's prove that $\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] < 0$:

$$\begin{aligned}
&= \frac{1}{w} - \frac{p_b}{w\left(\overline{g}+2p_b\right)} - \frac{2p_b}{\overline{g}\left(\overline{g}+2p_b\right)} = \\
&= \frac{2p_b(w - p_f d + \overline{g}) - p_b\overline{g} - 2p_b w}{w\overline{g}\left(\overline{g}+2p_b\right)} = \\
&= \frac{p_b\left(\overline{g} - 2p_f d\right)}{w\overline{g}\left(\overline{g}+2p_b\right)}
\end{aligned}$$

Since $\overline{g}(w)$ is an increasing function of w and $\overline{g}(\widehat{w}) = 2p_f d$, $\frac{p_b(\overline{g}-2p_f d)}{w\overline{g}(\overline{g}+2p_b)} < 0$ for all $w < \widehat{w} = \frac{2(p_f d)^2}{p_b} + p_f d$.

Using 10^8 Monte Carlo simulations it was proven that $\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] > 0$ for $p_b \in (0,1)$, $p_f d \in (0,1)$ and $w \in \left(p_f d, \frac{2(p_f d)^2}{p_b} + p_f d\right)$.

So, we proved that $\left[\frac{p_b}{(\overline{g}+2p_b)}\left(\frac{1}{w^2} + \frac{p_b}{w\overline{g}(\overline{g}+2p_b)} + \frac{4(\overline{g}+p_b)p_b}{\overline{g}^3(\overline{g}+2p_b)}\right) - \frac{1}{w^2}\right] > 0$ and $\left[\frac{1}{w} - \frac{p_b}{w(\overline{g}+2p_b)} - \frac{2p_b}{\overline{g}(\overline{g}+2p_b)}\right] < 0$ for $w < \widehat{w}$, or $\frac{\partial L_{s1}(w)}{\partial w} > \frac{\partial L_{s2}(w)}{\partial w}$.

QED.

B.8 Proof of Proposition 7

The equation (31) can be rewritten as:

$$\begin{aligned}
\frac{\partial U_{gov}(w,S)}{\partial S} &= -1 + \theta\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)} - \frac{\overline{g}+p_b}{w}\right] - \\
&\quad -\frac{\lambda\mu}{\sigma}\theta\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)} - \frac{\overline{g}+p_b}{w}\right]
\end{aligned}$$

If we show that $\left[\frac{\overline{g}}{(w-p_f d)} - \frac{(\overline{g}+p_b)}{w}\right] > 0$ for all $w < \widehat{w}$ then the Proposition will be proven.

$$\begin{aligned}
\frac{\overline{g}}{(w-p_f d)} - \frac{\overline{g}+p_b}{w} &= \frac{2p_b}{\overline{g}} - \frac{\overline{g}+p_b}{w} = \frac{2p_b\left(w-p_f d\right) + 2p_b p_f d - \overline{g}^2 - p_b\overline{g}}{\overline{g}w} = \\
&= \frac{\overline{g}^2 + p_b(2p_f d - \overline{g}) - \overline{g}^2}{\overline{g}w} = \frac{p_b(2p_f d - \overline{g})}{\overline{g}w}
\end{aligned}$$

Since $\overline{g}\left(w\right)$ is an increasing function of w and $\overline{g}\left(\widehat{w}\right) = 2p_f d$ (see the proof of *Proposition 1*), $\frac{p_b(2p_f d-\overline{g})}{\overline{g}w} > 0$ for all $w < \widehat{w}$.

Since $\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)}-\frac{(\overline{g}+p_b)}{w}\right]>0$, $\frac{\partial U_{gov}(w,S)}{\partial S}$ is a decreasing function of λ for all $w<\widehat{w}$.

QED.

B.9 Proof of Proposition 8

It was shown (the proof of Proposition 7) that

$$\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)}-\frac{\overline{g}+p_b}{w}\right]=\frac{p_b(2p_f d-\overline{g})}{(\overline{g}+2p_b)\,\overline{g}w}>0$$

for all $w<\widehat{w}$.

Since $\overline{g}\,(w)$ is an increasing function of w, $(\overline{g}+2p_b)\,\overline{g}w$ is an increasing function of w and $p_b(2p_f d-\overline{g})$ is a decreasing function of w. So, $\frac{p_b(2p_f d-\overline{g})}{(\overline{g}+2p_b)\overline{g}w}$ is a decreasing function of w and since $1-\frac{\lambda\mu}{\sigma}>0$ for all λ, μ, σ, $\frac{\partial U_{gov}(w,S)}{\partial S}$ (31) is a decreasing function of w.

QED.

B.10 Proof of Proposition 9

Firstly, we should prove that the equilibrium exists. Let $w(\lambda,S)$ be the equilibrium level of wages if the level of small business environment in the region is equal to λ, and the governor spends S on subsidies to small business. $w(\lambda,S)$ can be found from the equality $L_D\,(w)=L_{is}\,(w,\lambda,S)$. Since $L_D\,(w)$ is a decreasing function of w and $L_{is}\,(w,\lambda,S)$ is an increasing function of w, there is a unique solution $w\,(\lambda,S)$ of this equation. It is easy to show that $w\,(\lambda,S)$ is an increasing function of S (because $L_{is}\,(w,\lambda,S)$ is a decreasing function of S), so the higher subsidies to small business in the region, the higher the equilibrium level of wages. The derivative of the governor's utility function is equal to

$$\frac{\partial U_{gov}(w,\lambda,S)}{\partial S}=-1+\theta\left(1-\frac{\lambda\mu}{\sigma}\right)\frac{1}{(\overline{g}+2p_b)}\left[\frac{\overline{g}}{(w-p_f d)}-\frac{\overline{g}+p_b}{w}\right]$$

Let the initial level of subsidies be equal to 0 ($S=0$). If $\frac{\partial U_{gov}(w(\lambda,0),\lambda,0)}{\partial S}\leq 0$ then the governor does not want to increase subsidies and $S^E=0$ is the equilibrium level of subsidies.

If $\frac{\partial U_{gov}(w(\lambda,0),\lambda,0)}{\partial S}>0$ then the governor starts to increase S. Since $w\,(\lambda,S)$ is an increasing function of S and $\frac{\partial U_{gov}(w,\lambda,S)}{\partial S}$ negatively depends on w, $\frac{\partial U_{gov}(w(\lambda,S),\lambda,S)}{\partial S}$ is a decreasing function of S. So, the governor increases S until $\frac{\partial U_{gov}(w(\lambda,S),\lambda,S)}{\partial S}=0$ or $S=T$, which comes first. If $\frac{\partial U_{gov}(w(\lambda,S),\lambda,S)}{\partial S}=0$ then the further increase of S leads to a decrease of the governor's utility function (because $\frac{\partial U_{gov}(w(\lambda,S),\lambda,S)}{\partial S}$ is a decreasing function of S), so if there is $\overline{S}\leq T$, such as $\frac{\partial U_{gov}(w(\lambda,\overline{S}),\lambda,\overline{S})}{\partial S}=0$

then $S^E = \overline{S}$ is the equilibrium level of subsidies. If $\frac{\partial U_{gov}(w(\lambda,S),\lambda,S)}{\partial S} > 0$ for all $S \leq T$ then the governor increases subsidies until $S = T$ (he wants to increase subsidies more, but it is impossible because he cannot spends more than T). In this case $S^E = T$.

Now, we can prove that the equilibrium level of subsidies negatively depends on the level of small business environment, so if $\lambda_1 > \lambda_2$ then $S^E(\lambda_1) \leq S^E(\lambda_2)$.

There are three possible cases $S^E(\lambda_1) = 0$, $0 < S^E(\lambda_1) < T$ and $S^E(\lambda_1) = T$. Let's analyze all three cases.

$S^E(\lambda_1) = 0$.
Since $S^E(\lambda) \geq 0$ for each λ, $S^E(\lambda_1) \leq S^E(\lambda_2)$.

$0 < S^E(\lambda_1) < T$.
Since $0 < S^E(\lambda_1) < T \Rightarrow$

$$\Rightarrow \frac{\partial U_{gov}(w\left(\lambda_1, S^E(\lambda_1)\right), \lambda_1, S^E(\lambda_1))}{\partial S} = 0$$

Using the fact that $L_{is}(w,\lambda,S)$ is a decreasing function of λ ($L_{is}(w,\lambda_1,S) < L_{is}(w,\lambda_2,S)$ for each w), it is easy to show that $w(\lambda_1,S) > w(\lambda_2,S)$ for each S, particularly $w(\lambda_1,S^E(\lambda_1)) > w(\lambda_2,S^E(\lambda_1))$. It follows from *Proposition 8* that

$$\frac{\partial U_{gov}(w\left(\lambda_1, S^E(\lambda_1)\right), \lambda_1, S^E(\lambda_1))}{\partial S} < \frac{\partial U_{gov}(w(\lambda_2, S^E(\lambda_1)), \lambda_1, S^E(\lambda_1))}{\partial S}$$

Since $\frac{\partial U_{gov}(w,\lambda,S)}{\partial S}$ is a decreasing function of λ (*Proposition 7*),

$$\frac{\partial U_{gov}(w(\lambda_2, S^E(\lambda_1)), \lambda_2, S^E(\lambda_1))}{\partial S} > \frac{\partial U_{gov}(w(\lambda_2, S^E(\lambda_1)), \lambda_1, S^E(\lambda_1))}{\partial S}$$

So, it was shown that

$$\frac{\partial U_{gov}(w(\lambda_2, S^E(\lambda_1)), \lambda_2, S^E(\lambda_1))}{\partial S} > \frac{\partial U_{gov}(w\left(\lambda_1, S^E(\lambda_1)\right), \lambda_1, S^E(\lambda_1))}{\partial S} = 0$$

If $\frac{\partial U_{gov}(w(\lambda_2,S^E(\lambda_1)),\lambda_2,S^E(\lambda_1))}{\partial S} > 0$ then the governor wants to give subsidies more than $S^E(\lambda_1)$. Since $S^E(\lambda_1) < T$, the governor *can* increases subsidies, so if $\lambda_1 > \lambda_2$ and $0 < S^E(\lambda_1) < T$ then $S^E(\lambda_1) < S^E(\lambda_2)$.

$S^E(\lambda_1) = T$.
Since $S^E(\lambda_1) = T \Rightarrow$

$$\Rightarrow \frac{\partial U_{gov}(w(\lambda_1, T), \lambda_1, T)}{\partial S} \geq 0$$

Using *Proposition 7* and *Proposition 8* it is easy to show that

$$\frac{\partial U_{gov}(w(\lambda_2, T), \lambda_2, T)}{\partial S} > \frac{\partial U_{gov}(w(\lambda_1, T), \lambda_1, T)}{\partial S} \geq 0$$

It means that $S^E(\lambda_2) = S^E(\lambda_1) = T$, or $S^E(\lambda_2) \geq S^E(\lambda_1)$.

So, we show that in any case if $\lambda_1 > \lambda_2$ then $S^E(\lambda_1) \leq S^E(\lambda_2)$.
QED.